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PROPAGATION OF DISTURBANCES ABOVE THE FLOW DURING THE INTERACTION OF A HYPERSONIC FLOW WITH A BOUNDARY LAYER

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PROPAGATION OF DISTURBANCES ABOVE THE FLOW DURING THE INTERACTION OF A HYPERSONIC FLOW WITH A BOUNDARY LAYER

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ABSTRACT. It is shown that for a hypersonic flow with moderate or strong interaction, disturbances created, for instance, by a base section or some obstacle are propagated to the front edge of the body. No local areas with very large pressure gradients can be formed in the flow. It is then possible for zones of separation to form, having a length on the order of the body size and described, to the first approximation by boundary layer equations. From a mathematical point of view, the problem reduces to establishing the nonsingularity of the solution near the front edge, and finding proper solutions which satisfy the boundary conditions at the trailing end of the body. It is shown that, with weak interaction of the hypersonic flow with the boundary layer, there may be formed short areas of flows with free interaction and local-inviscous flows with large gradients of pressure, at the limits of which the disturbances can be transmitted above the flow. /40*

1. The general problem of steady motion of a body in a viscous and thermoconducting gas is described by Navier-Stokes equations. These equations are elliptical. Disturbances, created at some point of the flow, reach all other points, at least in principle.

However, at large Reynolds number, the generally accepted method of solving problems of aerodynamics is the application of equations for inviscous

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*Numbers in the margin indicate the pagination in the original foreign text.

gas (Euler equations) practically everywhere, with the exception of some narrow areas where allowance for viscosity is important. In particular, such areas are localized near surfaces of bodies (boundary layers). The equations for boundary layers are parabolic (Prandtl, 1904). With fixed boundary conditions, changes in equations in the area below the flow have no effect on the solution in the area above the flow. Hence, it was considered for a long time that at supersonic flow around bodies, in the absence of separation, the disturbances are not transmitted above the flow.

Still, in a number of cases, the boundary conditions are unknown beforehand, not only for the boundary layer but also for the inviscous external flow. They have to be determined by concurrent solutions, carried out simultaneously for both areas of the flow. Such a class includes, for instance, flows with "free interaction" [1,2] and with strong interaction [3, 4].

Although equations describing the flow in various areas prove to be hyperbolic and parabolic, the data on boundary conditions set below the flow should include also the area above the flow. A boundary condition which has to be determined by integration of equations has an integral character, and the problem becomes really of an integral-differential nature.

We have to note, however, that the information is not complete, as for elliptical equations. The class from which possible solutions are selected is more narrow. Solutions of this type have been obtained in [2, 5, 6].

Studies described in [2, 5, 6] dealt with supersonic flows ($M \sim 1, R \to \infty$, /41
where R - Reynolds number), in the area of free interaction around the point of separation on a smooth surface in front of the base section, and in the area of attachment of the zone of separation. The problem was solved through asymptotic analysis of solutions of complete Navier-Stokes equations, utilizing the known method of local asymptotic expansion. For these problems, the characteristic longitudinal dimension of the area in which disturbances penetrated above the flow amounted to $R^{-3/8}$, and the pressure disturbance was $\Delta p / p_1 \sim R^{-1/4}$.

Studies [2, 5, 6] have shown that the presence of free interaction signifies the transfer of disturbances above the flow; this fact leads to the necessity of previous selection of the appropriate method of solution.

It is of interest to consider flows in which the areas of free interaction are large, on the order of dimensions of the body. Such a type of problem is represented by a hypersonic flow of viscous gas around a slender body (for instance, a plane) with the value of the known interaction parameter $\chi = M_1 (d\delta / dx) \geqslant 1$. The known [3] self-modeling solution for strong interactions is valid only for a semi-infinite body; strictly speaking, it is applicable to finite bodies only for especially adopted boundary conditions at the trailing end of the body.

Another example is the problem of a blast of moderate intensity through the surface of a slender body in a supersonic gas flow. If we consider, as is done in a number of works, such as [7], that the velocity component of the gas normal to the surface of the body is small in comparison with the velocity of the incoming stream but, nevertheless, considerably exceeds the corresponding component in a viscous boundary layer, then in the first approximation we can utilize the model of a thin inviscous layer of the flow. The pressure distribution in a thin layer is determined by disturbances introduced by this layer into the initial supersonic flow. Because of a small thickness of the gas layer, the transfer of pressure across it is nonexistent in the first approximation, and the Euler equations degenerate into boundary layer equations without viscous terms. It can be shown for the same problem that the effect of the trailing end of the body should be considered along the whole body, up to the leading edge.

2. We shall consider the flow of a hypersonic stream of a viscous thermoconducting gas around a slender body. The Reynolds number calculated along the length of the body l, velocity of the incoming stream u_1, characteristic density value ρ_2, and the viscosity coefficient μ_2 at the surface of the body is $R_2 \gg 1$. Then, the disturbed area of the flow can be divided into an inviscous flow and the boundary layer of thickness $\delta \sim l R_2^{-1/2}$.

Further, we shall consider that the thickness of the body δ_2 is of the same order as, or even smaller than, δ (for instance, $\delta_2 = 0$ — a membrane or a thread). In this case, the pressure distribution on the outer limit of the boundary layer depends on changes of thickness of the boundary layer displacement and, at the same time, it greatly affects the distribution of the thickness of displacement. This is flow with free interaction. The extent of the flow area with free interaction depends substantially on values of the numbers M_1 and R_2.

In supersonic flows $(M_1^2 - 1)^{1/2} \sim 1$ the size [2, 5, 6] is on the order of $lR_2^{-3/8}$. We shall note that at $M_1 \sim 1$ the pressure gradient induced by the boundary layer for specific points of flow (point of separation, attachment, etc), influences the boundary layer only to the second approximation. Hence, there is no free interaction in the larger part of the body.

We shall consider the flow with free interaction at $M_1 \to \infty$ for characteristic cases of flow around a corner point or a base section, and also the points of separation and attachment. For all these flows at moderate supersonic velocities, the scales of coordinates, flow functions, equations, and boundary conditions were the same (with the exception of the initial and final condition). We shall show that, at small values of the known parameter of hypersonic interaction $\chi = M_1 / R_2^{1/2} \ll 1$, the situation is the same for hypersonic flows. /42

Let us consider first the flow near the point of separation on a flat plate at $M_1 \gg 1$. Let the gas enthalpy at the surface of the body be of the same order of magnitude as the enthalpy of the drag (although it may amount, for instance, to 0.1 of the latter). If far from the point of separation $\chi \ll 1$, then the pressure gradient induced by weak interaction is $(l / p_1)\partial p / \partial x \sim \chi$. Following [2], we shall estimate values of the pressure gradient which should exist near the point of separation, if the latter is located the distance l from the leading edge. In the main part of the boundary layer $\delta / l \sim R_2^{-1/2}$

$$u \sim u_1, \quad \frac{\Delta u}{u_1} \sim \frac{\Delta \rho}{\rho_1} \sim \frac{\Delta p}{p_1}, \quad \Delta n \sim \frac{\Delta p}{p_1} R_2^{-1/2} l$$

Here Δn — is the change of thickness of the flowlines. At the wall, the velocity is zero. Hence, near the surface of the body, in accordance with equations of the motion

$$\Delta u_3 \sim u_3, \quad \frac{\Delta u_3}{u_1} \sim \left(\frac{\Delta p}{p_1}\right)^{1/2}$$

Let δ_3 be a thickness of the area 3 in which $\Delta u_3 \sim u_3$; then, $u_3 \sim u_1 \times \times (\delta_3 / l) R_2^{-1/2}$. The thickness of this area changes by the order of magnitude

$$\frac{\delta_3}{l} \sim \left(\frac{\Delta p}{p_1}\right)^{1/2} R_2^{-1/2} \gg \frac{\Delta n}{l}$$

It means that the displacement thickness of the whole boundary layer in the main term changes by δ_3 / l. Then, in accordance with the hypersonic theory of small disturbances (for $\Delta p / p_1 \ll 1$) we obtain $\Delta p / p_1 \sim (\delta_3 / x_3) M_1$, where x_3 — the unknown longitudinal dimension of the perturbed area. If the flow streams in the area 3 before the point of separation are to pass into the area where the pressure increases, there should be in the area 3 at least some significant viscosity forces: $\rho_2 u_3^2 / x_3 \sim \mu_2 u_3 / \delta_3^2$. This last condition makes the system of interrelations for the scale $\Delta p / p_1, u_3, \delta_3, x_3$ closed; it enables us to obtain the following relations at $\chi \ll 1$:

$$\delta_3 / l \sim M_1^{1/4} R_2^{-5/8} = \chi^{1/4} R_2^{-1/2}, \qquad x_3 / l \sim \chi^{3/4} = M_1^{3/4} R_2^{-3/8} \tag{2.1}$$

$$u_3 / u_1 \sim M_1^{1/4} R_2^{-1/8} = \chi^{1/4}, \quad \Delta p / p_1 \sim M_1^{1/2} R_2^{-1/4} \sim \chi^{1/2}, \quad \Delta n / l \sim \chi^{1/2} R_2^{-1/2} \tag{2.2}$$

Utilizing the methods in [2], we can easily obtain the complete system of equations and boundary conditions, closing the problem. They are fully analogous to those obtained in [2] if we replace $\varepsilon = R_1^{-1/2}$ by $\chi = M_1 / R_2^{1/2}$. It follows from this that the theory developed for $M_1 \sim 1$ is limiting for hypersonic velocities at $\chi \to 0$.

Fully analogous results can be easily obtained for flows in the area of attachment, and near the base section, by following [5] or [6]. The characteristic feature of these flows is that the propagation of perturbations above the flow occurs at short distances only, with the formation of areas having large gradients of pressure. Further increase of the pressure gradient may lead to the appearance of locally inviscous areas of flow. Locally inviscous flow around the base section was investigated in [8].

3. The situation changes basically if $O(\chi) \to O(1)$. It is apparent from /43
(2.1) that the scale of the area of free interaction $x / l \to O(1)$. And it follows from (2.2) that the division of the boundary layer into subareas of different scale disappears, since $\Delta n / l \sim \delta_0 / l$.

It follows also from $\Delta p / p \sim \chi^{1/2} \to 1$. In this way, the whole boundary layer begins to participate in the process of free interaction.

We shall show that, at $\chi \gtrsim 1$, there can be no region of flow in which the pressure gradient is larger in order of magnitude than the gradient induced on the body by free interaction.

Let us assume that near the base section or in front of the point of separation there is a drop of pressure $\Delta p \lesssim p$ (we are not considering the case of $\Delta p / p \gg 1$, since in flows of rarefaction it is impossible, and in flows of compression such values of $\Delta p / p$ do not occur because of the displacement of the point of separation above the stream, as will be shown below).

If the induced gradient of pressure is larger by an order of magnitude than the initial one, then the dimensions of the perturbed area Δx should satisfy the condition $\Delta x / l \ll 1$. For $M_1 \gg 1$ and $\chi \gtrsim 1$ in the initial boundary layer

$$p \sim \rho_1 u_1^2 \tau^2, \qquad \rho \sim \rho_1 \tau^2, \qquad (d\delta / dx)_0 \sim \tau$$

In accordance with the equation of motion ($\rho u u_x \sim p_x$) disturbances of velocity and pressure are interconnected by the relation

$$u \sim \Delta u \sim (\Delta p / \rho_1 \tau^2)^{1/2}$$

This estimate is also valid for $\Delta p / p_1 \ll 1$, since near the surface of the body one can always find a layer in which velocity perturbations are of the same order as the initial velocity. Just as for flows with weak interaction, the complete change in thickness of the boundary layer is equal in magnitude to the thickness of the layer in which the velocity changes by an order of magnitude. For the thickness of this layer, we have the relations

$$y \sim \frac{u}{u_1} \tau l, \qquad \Delta\delta \sim l \left(\frac{\Delta p}{\rho_1 u_1^2} \right)^{1/2}$$

Since $\Delta p / p_1 \lesssim 1$, then $\Delta\delta / \Delta x \lesssim \tau$. Utilizing the external boundary condition and the obtained estimates

$$\frac{\Delta p}{\rho_1 u_1^2} \sim \Delta \left(\frac{d\delta}{dx} \right)^2 = \left[\left(\frac{d\delta}{dx} \right)_0 + \frac{\Delta\delta}{\Delta x} \right]^2 - \left(\frac{d\delta}{dx} \right)_0^2 \sim \left(\frac{d\delta}{dx} \right)_0 \frac{\Delta\delta}{\Delta x} \sim \tau \frac{\Delta\delta}{\Delta x}$$

$$\frac{\Delta\delta}{\Delta x} \sim \frac{l}{\Delta x} \left(\frac{\Delta p}{\rho_1 u_1^2} \right)^{1/2}, \qquad \frac{\Delta x}{l} \sim \tau \left(\frac{\rho_1 u_1^2}{\Delta p} \right)^{1/2} \gtrsim 1$$

But the dimension of the perturbed area cannot be larger than the characteristic dimension of the body. It follows from this, therefore, that $\Delta x \sim l$.

In this connection, the solution of the problem for a boundary layer under conditions of free (moderate or strong, $\chi \geqslant 1$) interaction cannot be fully determined by the initial conditions and the boundary conditions on the outer edge of the boundary layer and on the body surface. There should be also branches of the solution making it possible to satisfy the boundary condition at the trailing end of the body which, as will be shown below, can be imposed, for instance, upon the pressure value.

In conformity with the usual estimates for a hypersonic boundary layer at $\chi \geqslant O(1)$ we take the following coordinates and asymptotic expressions for the flow function: /44

$$x = x^\circ / l^\circ, \quad y = y^\circ / l^\circ\tau, \quad \tau = (\mu_2^\circ / \rho_1^\circ u_1^\circ l^\circ)^{1/4}$$
$$u^\circ(x^\circ, y^\circ, M_1, R_1) = u_1^\circ[u(x, y) + \ldots], \quad v^\circ(x^\circ, y^\circ, M_1, R_1) = $$
$$= u_1^\circ\tau[v(x, y) + \ldots]$$
$$p^\circ(x^\circ, y^\circ, M_1, R_1) = p_1^\circ\gamma M_1^2\tau^2[p(x, y) + \ldots], \quad \rho^\circ(x^\circ, y^\circ, M_1, R_1) =$$
$$= \rho_1^\circ\tau^2[\rho(x, y) + \ldots]$$
$$H_0(x^\circ, y^\circ, M_1, R_1) = (u_1^\circ)^2[H(x, y) + \ldots], \quad \mu^\circ(x^\circ, y^\circ, M_1, R_1) =$$
$$= \mu_2^\circ[\mu(x, y) + \ldots] \tag{3.1}$$

Here the degree signs at the top signify dimensional quantities.

Substituting (3.1) into the complete Navier-Stokes equations and making the limiting transition $M_1 \to \infty$, $R_1 \to \infty$ at $\chi \gtrsim O(1)$ we obtain the equations

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$
$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \quad p = \frac{\gamma - 1}{\gamma}\left(H - \frac{u^2}{2}\right)\rho$$
$$\rho\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\mu}{\sigma}\frac{\partial H}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu\left(1 - \frac{1}{\sigma}\right)\frac{\partial}{\partial y}\frac{u^2}{2}\right] \tag{3.2}$$

The boundary conditions are

$$u(x, 0) = v(x, 0) = 0, \qquad H(x, 0) = H_2$$
$$u(x, \delta) = 1, \quad v(x, \delta) = d\delta / dx, \quad H(x, \delta) = 1/2 \tag{3.3}$$

Here, the outer edge of the boundary layer $y = \delta$ is precisely defined, $\rho(x, \delta) = \infty$, since the flow rate of gas in the boundary layer is negligibly small in comparison with the flowrate in the inviscous area of perturbed flow, whereas the thicknesses may be of the same order. It is just this difference in the flowrate order of magnitude that enables us to develop the correct theory of the boundary layer.

For the solution of the problem on a computer, it is convenient to introduce the following variables:

$$\xi = \int_0^x \rho_2\, dx, \quad \eta = (2\xi)^{-1/2} \int_0^y \rho\, dy$$
$$u = f'(\xi, \eta), \quad g = 2H, \quad \psi(x, y) = \sqrt{2\xi} f(\xi, \eta) \tag{3.4}$$

Then the equations assume the usual form

$$(Nf'')' + ff'' - \beta(\xi)(g - f'^2) = 2\xi(f'f'^{\cdot} - f''f^{\cdot})$$
$$\left(\frac{N}{\sigma} g'\right)' + fg' + \frac{1}{2}\left[N\left(1 - \frac{1}{\sigma}\right) f'f''\right]' = 2\xi(f'g^{\cdot} - f^{\cdot}g') \tag{3.5}$$

The boundary conditions are /45

$$f(\xi, 0) = f'(\xi, 0) = 0, \quad f'(\xi, \infty) = 1, \quad g(\xi, 0) = g_2, \quad g(\xi, \infty) = 1$$
$$v(\xi, \infty) = \frac{d\delta}{dx}, \quad N = \frac{\rho\mu}{(\rho\mu)_2}, \quad \rho = \frac{2\gamma}{(\gamma - 1)} \frac{p}{(g - f'^2)}$$
$$\mu = \left[\frac{(g - f'^2)}{g_2}\right]^{\omega}, \quad \beta(\xi) = \frac{\gamma - 1}{\gamma} \frac{d \ln p}{d \ln \xi}$$
$$v_1(\xi) = v(\xi, \infty) = \frac{p(\xi)}{g_2} \frac{d}{d\xi}\left[\frac{\sqrt{2\xi}}{p(\xi)} \int_0^\infty (g - f'^2)\, d\eta\right] \tag{3.6}$$

The indices 1 and 2 signify parameter values on the outer side of the boundary layer and at the wall, respectively. For the calculation, it is necessary to consider the flow in an inviscous shock layer. Following the hypersonic theory of small perturbations, as in [3], we shall introduce the following functions and coordinates for the shock layer:

$$x^\circ = l^\circ x, \quad y^\circ = \tau l^\circ y, \quad u^\circ = u_1^\circ [1 + \tau^2 U(x, y, \chi) + \ldots]$$
$$p^\circ = \gamma p_1^\circ M_1^2 \tau^2 P(x, y, \chi) + \ldots, \quad \rho^\circ = \rho_1^\circ R(x, y, \chi) + \ldots,$$
$$v^\circ = \tau u_1^\circ V(x, y, \chi) + \ldots \tag{3.7}$$

The system of equations and boundary conditions is

$$\frac{\partial R}{\partial x} + \frac{\partial RV}{\partial y} = 0, \quad R\left(\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y}\right) + \frac{\partial P}{\partial y} = 0$$
$$\left(\frac{\partial}{\partial x} + V\frac{\partial}{\partial y}\right)\left(\frac{P}{R^\gamma}\right) = 0, \quad V[x, \delta(x)] = \frac{d\delta}{dx}$$
$$\gamma P[x, g(x)] = \frac{2\gamma\chi^2}{\gamma+1}\left(\frac{dg}{dx}\right)^2 - \frac{\gamma-1}{\gamma+1}$$
$$R[x, g(x)] = \left[\frac{\gamma-1}{\gamma+1} + \left(\frac{2}{\gamma+1}\right)\frac{1}{\chi^2}\left(\frac{dg}{dx}\right)^{-2}\right]^{-1}$$
$$V[x, g(x)] = \frac{2}{\gamma+1}\frac{dg}{dx}\left\{1 - \frac{1}{\chi^2}\left(\frac{dg}{dx}\right)^{-2}\right\} \tag{3.8}$$

where y = g(x) — equation for the form of the shock wave.

For practical calculations, sufficiently accurate results are obtained by the tangential wedge method

$$p(\xi) = \frac{1}{\chi^2\gamma} + \frac{\gamma+1}{4}v_1^2(\xi) + v_1(\xi)\left\{\frac{1}{\chi^2} + \left[\frac{\gamma+1}{4}v_1(\xi)\right]^2\right\}^{1/2} \tag{3.9}$$

It is necessary to remark here that at $\chi \to \infty$, the parameter χ vanishes both from (3.8) and (3.9). The complete problem for dimensionless variables ceases to depend on χ.

We shall clarify now the character of the nonsingularity of solution at different values of the interaction parameter χ.

Let the boundary condition determining the necessary branch of solution be set at $x^\circ = l^\circ$, i.e., at $x = x_\circ = 1$. If the point of separation lies there, then we have $f_2'' = 0$. If corner point 0 corresponds to it, then the pressure on it is the given pressure.

We shall show at first that if we managed to find at least one solution for which, for instance, $f_2''(x_\circ) = 0$, then there is also a group of transformations which allow us to satisfy the condition $x_\circ = 1$. For this purpose we shall introduce the transformation /46

$$\xi = b\xi_4, \quad \eta = \eta_4, \quad p = \frac{1}{b} p_4, \quad \rho = \frac{\rho_4}{b}, \quad u = u_4, \quad h = h_4, \quad v_1 = \frac{1}{\sqrt{b}} (v_1)_4$$
$$y = b^{3/2} y_4, \quad x = b^2 x_4, \quad \chi = \sqrt{b}\chi_4, \quad f = f_4, \quad g = g_4 \tag{3.10}$$

where b is the undetermined constant, and for (3.8)

$$R = R_4, \quad g = b^{3/2} g_4, \quad P = \frac{1}{b} P_4, \quad V = \frac{1}{\sqrt{b}} V_4 \tag{3.10a}$$

With this change of variables, all equations and boundary conditions expressed in terms of new variables have the same form as (3.4)-(3.6) and (3.8), (3.9) in old variables. The only difference lies in the fact that the value of χ_4 is known beforehand, and the coordinate x_4 of a singular point is not given. Setting χ_4 arbitrarily, we obtain a solution (numerically or in series) with a singular point at some value $x_{40}(\chi_4)$. In accordance with Formulas (3.9) and the condition $x_0 = 1$ we obtain

$$1 = b^2 x_{40}(\chi_4), \qquad \chi = \sqrt{b}\chi_4 \tag{3.11}$$

Then, the first formula gives us b, while the second — provides a value of χ corresponding to the given χ_4

$$\chi = \chi_4 [x_{40}(\chi_4)]^{-1/4} \tag{3.12}$$

If $\chi_4 \gg 1$, then χ_{40} does not depend on χ_4, and Formula (3.11) gives a solution for direct as well as reverse problems. If, however, $\chi_4 \sim 1$, then we can solve without adjustment only the reverse problem of finding χ at a given χ_4. For solution of the direct problem (in which χ is given), it is necessary to choose χ_4 such that the condition (3.12) is satisfied.

The existence of a group of transformations (3.10) demonstrates the fact that, at least at $\chi = \infty$, there cannot be nonself-modeling solutions, which are not terminated by a singular point (for instance, $p_1 = 0$) or place, where boundary conditions are changing, and which continue for all ξ from 0 to ∞ when the given boundary conditions are maintained. As a matter of fact, if such a solution existed, then in the presence of (3.10) it could be transformed into a self-modeling solution, but it is defined in a unique manner. Thus, in order to establish the nonsingularity and existence of solutions of the requisite type, it is sufficient to demonstrate the existence of nonself-modeling solutions even at $\xi \ll 1$.

In the neighborhood of $\xi = 0$, the indeterminacy of the solution can be established by means of a coordinate expansion. Using a computer method analogous to that developed in [2],the solution of the problem can be extended to actual finding various branches of solution at finite values of ξ up to the corresponding singular points.

It is simplest to establish the indeterminacy of the solution for $\chi = \infty$, since then it is sufficient to show that, in addition to the known self-modeling solution, there is also a nonself-modeling one. Further, we shall utilize (3.9) and $\chi = \infty$

$$f(\xi,\eta)=f_0(\eta)+\xi^{a+1}\frac{A_1}{A_0}f_1(\eta)+\ldots,\quad g(\xi,\eta)=g_0(\eta)+\xi^{a+1}\frac{A_1}{A_0}g_1(\eta)+\ldots$$
$$p(\xi)=\frac{A_0}{\xi}+A_1\xi^a+\ldots,\qquad v_1(\xi)=\frac{B_0}{\sqrt{\xi}}+B_1\xi^{a+1/2}+\ldots \tag{3.13}$$

Substituting (3.13) in the initial equations and boundary conditions, we /47
obtain

$$(N_0 f_0'')' + f_0 f_0'' + \frac{\gamma-1}{\gamma}(g_0 - f_0'^2) = 0$$

$$\left(\frac{N_0}{\sigma} g_0'\right)' + f_0 g_0' + \frac{1}{2}\left[N_0\left(1-\frac{1}{\sigma}\right) f_0' f_0''\right]' = 0$$

$$N_0 = [(g_0 - f_0'^2)/g_2]^{\omega-1},\ f_0(0) = f_0'(0) = 0,\ g(0) = g_2,\ f_0'(\infty) = 1,\ g_0(\infty) = 1$$

$$B_0 = \frac{3}{\sqrt{2}} D F_0, \quad A_0 = \frac{\gamma+1}{2} B_0^2, \quad F_0 = \int_0^\infty (g_0 - f_0'^2)\, d\eta, \quad D = g_2^{-1} \tag{3.14}$$

This part represents the known self-modeling solution for strong interaction.

$$(N_0 f_1'' + N_1 f_0'')' + f_0 f_1'' + f_0 f_1'' + \frac{\gamma-1}{\gamma}(g_1 - 2 f_0' f_1') -$$

$$- 2(a+1)(f_1' f_0' - f_0'' f_1) = \frac{\gamma-1}{\gamma}(a+1)(g_0 - f_0'^2)$$

$$\left(\frac{N_0}{\sigma} g_1' + \frac{N_1}{\sigma} g_0'\right)' + f_1 g_0' + f_0 g_1' + \frac{1}{2}\left[N_0\left(1-\frac{1}{\sigma}\right) f_0' f_1'' + \right. \tag{3.15}$$

$$\left. + N_0\left(1-\frac{1}{\sigma}\right) f_1' f_0'' + N_1\left(1-\frac{1}{\sigma}\right) f_0' f_0''\right]' - 2(a+1)(f_0' g_1 - f_1 g_0') = 0$$

$$N_1 = (\omega-1) N_0 \frac{g_1 - 2 f_0' f_1'}{g_0 - f_0'^2}, \quad f_1(0) = f_1'(0) = f_1'(\infty) = g_1(0) = g_1(\infty) = 0.$$

$$A_1 = (\gamma+1) B_0 B_1, \quad B_1 = D\left[F_1 \frac{5+2a}{\sqrt{2}} - F_0 \sqrt{2}(a+1)\right] \frac{A_1}{A_0}$$

$$F_1 = \int_0^\infty (g_1 - 2 f_0' f_1')\, d\eta \tag{3.16}$$

The system of Equations (3.15) is linear, but nonhomogeneous at $a > -1$ [$a < -1$ makes no sense in view of (3.13)].

The linear homogeneous system (3.16) for coefficients A_1, B_1 has a nontrivial solution only in the case where its determinant is equal to zero. This provides the condition for determining the parameter a

$$a = [10F_1(a) - 7F_0] / 4[F_0 - F_1(a)]$$

Numerical calculations give $a \approx 49.6$ for $\omega = \sigma = g_2 = 1$. Let us assume that a for the requisite range has been found. Then A_1, B_1 can be determined within an accuracy of an arbitrary constant factor. Its magnitude determines the value of x_0, the coordinates of the singular point and its sign gives the choice of the type of singular point. The given transformations lead to the following form of the similarity rule for the coefficient of pressure at the point of separation:

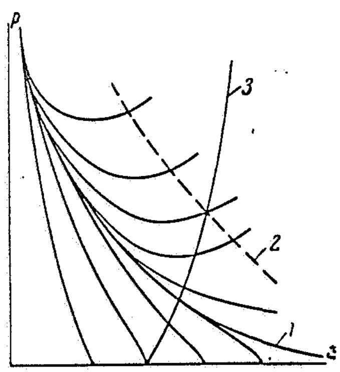
$$C_p = 2\left(\frac{\mu_2{}^0}{\rho_1{}^0 u_1{}^0 l^0}\right)^{1/4} p_{40} x_{40}^{1/2}$$

where p_{40}, x_{40} - functions of g_2; γ, and χ can be obtained by numerical solution of the problem. At $\chi \to \infty$, the dependence on χ vanishes, and the second limit for C_p is obtained, apart from the one found in [2] for $\chi = 0$. /48

The preceding results lead to an important conclusion for the asymptotic theory of separated flows if the separation and attachment of the flow occur on a smooth surface as, for instance, in the case of separation in front of a panel, not beginning at the leading edge. If separation does not start at the leading edge then, as in the case of $M \sim 1$, the pressure in the whole area of separation and the ratio of the longitudinal dimension to the transverse dimension are of the same order of magnitude as in the area of free interaction near the point of separation. But in the given case $M_1 \to \infty, \chi \gtrsim 1$ the characteristic longitudinal dimension of the area of free interaction is of the same order of magnitude as the length of the body, as has been shown. From this, it follows that the flow in the whole zone of separation (the length of which cannot be larger than the body dimensions) is described by these same equations of the boundary layer in the first approximation. This holds everywhere with the exception of small zones on the order of the boundary layer thickness (vicinity of corner points, etc.). This means that, until the point of separation reaches the leading edge, the angle of inclination of a panel should be on the order of $O(\tau)$. If it is larger, the separation should begin from the leading edge. At $M_1 \sim 1$ the areas of free interaction have the length $O(R_1^{-3/8})$ [2]. Hence, even at angles of inclination of panel O(1) the separation may not begin from the leading edges, and equations of flow inside the separated zone do not reduce to the Prandtl equations [6], even in the first approximation.

We note that Formula (3.10) determines the similarity rule for $\chi = \infty$, valid for flows of compression and rarefaction, which will be considered below

4. We shall carry out briefly a qualitative analysis of the results obtained for the case of $\chi = \infty$, using a scheme of integral curves of the Problem (3.5), (3.6) on the $p\xi$ plane (Figure). Line 1 represents the known self-modeling solution (3.14). Below this line, there is a family of non-self-modeling solutions corresponding to $A_1 < 0$ (3.13), at which $p = 0$ for finite values of ξ, since $p < 0$ has no physical sense, and $p \to 0$ at $\xi \to \infty$ is impossible because of the existence of the group (3.10), as was shown above. Above the line 1 in the figure there is a family of curves corresponding to $A_1 > 0$, and to the decrease of f_2'' to zero along the dotted curve 2. This is the line of separation. For each curve, one can calculate $x(\xi)$ from the Formulas (3.4) and (3.6). Line 3, along which $x = 1$, has an additional boundary condition for flow on a body with finite length and a flat surface. The intersection of lines 2 and 3 determines the value of the base pressure, corresponding to the flow separation ($f_2'' = 0$) just near the base section. If the base pressure coincides with the value of p at the intersection point of curves 1 and 3, then the distribution of pressure over the body is the same as in the self-modeling solution. A part of the curve 3 lying above the line 2 corresponds to the location of the separation point above the flow from the base section. As was pointed out above, to obtain the whole family of curves for both the nonself-modeling solutions,



it is sufficient to find one curve for each, and the remaining ones are obtained from (3.10). Then, to each value of the base pressure there corresponds one integral curve, on which this base pressure is reached on line 3. We shall note that at $(p - p_0)/p_0 = \varepsilon << 1$, where $p_0 = A_0\xi^{-1}$, Formula (3.13) gives a rigorous first approximation relative to ε for distribution of the parameters on the whole body to $x = 1$.

As was remarked earlier, for the case of $\chi \sim 1$ or for a more complex form of the body, adjustments are necessary.

One more consequence arises from results of the numerical solution of (3.15). Since the value of a in (3.13) is large according to the calculations performed ($a \sim 50$ for $\omega = \sigma = g_2 = 1$), the deviation of the integral curves from the self-modeling solution is hardly noticeable, and then occurs very sharply. This feature explains why, in the application of insufficiently /49
accurate integral methods for solving problems with free interaction — for instance, the Crocco-Lies method — it is necessary to introduce the concept of subcritical and supracritical boundary layers, and a step-wise transition from the regime at which there is no transfer of perturbations above the stream, to the regime with the transfer of disturbances above the flow.

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